resolution instrument have dimensions, Δx and Δy, with reference at x_0, y_0. The following definition of symbols applies hereinafter:

x, y : the coordinates of the imaged surface; positive real numbers.

$\Delta x, \Delta y$: pixel dimensions of the higher resolution image; positive real numbers.

x_0, y_0 : the lower left corner of the imaged surface (the zero point of the coordinate system).

$x_i = x_0 + i\Delta x$ and $y_j = y_0 + j\Delta y$ refer to the i, j pixels of higher resolution image.

M, N refers to the higher resolution instrument pixel elements.

m, n refers to the lower resolution instrument pixels.

(i,j) refers to the higher resolution image.

(i', j') are the labels for the lower resolution instrument.

R_x, R_y are positive numbers defining the resolution of the higher resolution instrument along the two coordinates x, y.

Using the above, the pixels in the higher resolution instrument are identified as follows:

(1) $(i\,j),\ i = 0, 1, \cdots M-1,\ j = 0, 1, \cdots N-1$

For the lower resolution instrument, the pixels are identified as:

(2) $(i'j'),\ i' = 0, 1 \cdots \frac{M}{m} - 1,\ j' = 0, 1 \cdots \frac{N}{n} - 1$

Where $\frac{M}{m}$ and $\frac{N}{n}$ are the ratios between the coarser and finer resolution cells.

LOWER CASE PRINTED LETTER a.

With the identification of the geometric relationship between the two-dimensional pixel array of the higher resolution instrument and imaging surface, the complex value of the image is:

$$C_{ij} = \int\int h(x - x_i, y - y_i)\, a(x,y) \exp(i\varnothing(x,y))\, dxdy \quad \text{(higher resolution)}$$

$\int_{a}^{a} \int_{-\infty}^{+\infty}$

The image intensity is:

$$I_{ij} = |C_{ij}| \quad \text{(higher resolution)}$$

i j

(3) The expected value of image intensity is:

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$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}$$

$$\langle I_{ij}\rangle = \int\int |h(x-x_i, y-y_j)|^2 \langle [a(x,y)]^2\rangle dxdy \quad \text{(higher resolution)}$$

From equation (2), the coordinates of the pixels of the lower resolution instrument are identified; which leads to:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}$$

(4) $x_i' = x_0 + \frac{(m-1)\Delta x}{2} + i' m \Delta x$

(5) $y'_{j'} = y_0 + \frac{(n-1)\Delta x}{2} + j' n \Delta y$

These two equations assume that the two images are co-registered. From this, the expected intensity of the lower resolution instrument is obtained:

(6) $\langle I_{i'j'}\rangle = \int\int |h'(x-x_i', y-y_j')|^2 \langle [a(x,y)]^2\rangle \; dxdy$

The two images of the same scene are statistically independent, and the relative variance of the two images are:

(7) $v = \frac{\langle (I_{ij} - \langle I_{ij}\rangle)^2\rangle}{(\langle I_{ij}\rangle)^2}$ Higher Resolution Image (independent of i j)

(8) $v = \frac{\langle (I_{i'j'} - \langle I_{i'j'}\rangle)^2\rangle}{\langle (I_{i'j'})\rangle^2}$ Lower Resolution Image (independent of i' j')

The square of the absolute value of the impulse responses of the two instruments are:

(9) $|h(x,y)|^2 = \frac{1}{RxRy}$ for $|x| \le Rx, |y| \le Ry$ Higher Resolution Instrument

(10) and $|h'(x,y)|^2 = \frac{1}{mn} |h(x/m, y/m)|^2$ Lower Resolution Instrument

By combining (3), (6), ~~(9) and (10)~~, the result is

$$\sum_{i=i'm}^{(i'+1)m-1} \sum_{j=j'n}^{(j'+1)n-1}$$

(11) $\langle I_{i'j'}\rangle = \frac{1}{mn}\left(\sum_{i=i'm}^{(i'+1)m-1} \sum_{j=j'n}^{(j'+1)n-1} \langle I_{ij}\rangle\right)$

That is, the expected intensity of the lower resolution image is the local average of the higher resolution image. The fusion of the two image intensities is obtained as the solution to the following minimization problem:

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$$\sum_{i=0}^{M-1} \sum_{j=0}^{N-1}$$

(12) $$\begin{Bmatrix} min \\ X_{ij} \end{Bmatrix} \left(\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \right) \left(X_{ij} - I_{ij} \right)^2$$

$$\sum_{k=i'm}^{(i'+1)m-1} \sum_{l=j'n}^{(j'+1)n-1}$$

Subject to the constraints:

(13) $$\frac{\displaystyle\sum_{k=i'm}^{(i'+1)m-1} \sum_{l=j'n}^{(j'+1)n-1} X_{kl}}{mn} = \frac{\frac{1}{v'} I_{i'j'} + \frac{1}{v} \displaystyle\sum_{k=i'm}^{(i'+1)m-1} \sum_{l=j'n}^{(j'+1)n-1}}{\frac{1}{v} + \frac{mn}{v}}$$

BOTH SUM LIMITS IN EQ. 13 ARE THE SAME

And:

(14) $X_{ij} \geq 0$

Where, in (13) i=0, 1--M/m-1, j=0, 1---- N/n-1 - and in (14) i=0,1, --M-1 and j=0,1, ---N-1

Equation (13) enforces the condition that the expected image intensity of the coarser resolution instrument must be the appropriate weighted average (with weights $\frac{1}{v'}$ and $\frac{mn}{v}$ of the coarser resolution estimated from the two images. Equation (14) indicates that the intensity is always positive. The minimization problem with MN independent variables can be decomposed in MN/mn independent variables. The sub-problems are labeled: i' = 0, 1, --- M/m - 1, j' = 0, 1, ---N/n -1. The (i', j')th sub-problem is the following:

$$\sum_{k=i'm}^{(i+1)m-1} \sum_{l=j'n}^{(j'-1)n-1}$$

(15) $$\frac{mm}{X_{kl}} \left(\sum_{K=im}^{(i+1)m-1} \sum_{l=j'n}^{(j'-1)n-1} \right) \left(X_{kl} - I_{kl} \right)^2$$

Subject to the constraints of (13) and

(16) $X_{kl} \geq 0$

Where, in (16) : k = i'm, i'm + 1, ---(i' + 1) m - 1

l = j'n, j'n + 1 --- ('j '+ 1) -1

By thusly performing a fusion of the images as taught by Costantini et al, a better match between the sensors can be achieved.